Transfer Matrix Modelling for the 3-Dimensional Vibration Analysis of Piping System Containing Fluid Flow

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For three dimensional vibration analysis of piping system containing fluid flow, a transfer matrix formulation is presented. The fluid velocity and pressure were considered, that coupled to longitudinal and flexural vibrations. Transfer matrices were derived from direct solutions of the differential equations of motion of pipe conveying fluids, and the variations of natural frequency with flow velocities for straight and curved pipes were investigated. The results were confirmed to the corrections of known data. The scheme of this study can be easily applied to the related fields, using small size personal computers with core memory about 200kbytes.

Key Words : Transfer Matrix, State Vector, Vibration of Piping System, Critical Flow Velocity

1. Introduction

Pipe lines are employed in industrial applications such as petroleum and chemical plants, fuel lines of transportations, heat exchangers, and so on. The function of pipe lines is so significant that they might be considered as blood circulation of human body. Inside of pipe lines, fluid flows and effects on dynamic behaviors of the piping systems, which play a great role in the stability and the efficiency of the systems. Accordingly, accurate predictions of dynamic behaviors of the piping systems are indispensable to prevent the undesirable response and to enhance the safety. Therefore, many researchers have been interested in the study on modelling techniques for the dynamic analysis. Housner (1952) is one of beginners who endeavored dynamic analysis of pipe lines with fluid flow. He studied on the vibration characteristics of oil pipe line by modelling as a simply supported beam. Benjamin (1961) and Gregory and Paidossis (1966) modelled cantilevered pipe as a branch pipe and an elastic continuum pipe, respectively, to investigate the dynamic behaviors. For curved pipes, the effect of curvature had been neglected in Kellogg Co. 's book (1956). Unny et. al. (1970) studied on the effect of curvature on the dynamic behaviors of curved pipes, and Chen (1972, 1973) analyzed curved pipes by investigating in-plane and out-of-plane vibrations. In practice, since the geometry of piping systems and the fluid effects are very complicated, analytic methods are not sufficient for the accurate analysis of real piping systems. For those problems, it is desirable to use numerical methods such as finite element method and transfer matrix method. Kohli and Nahera (1984) presented two dimensional analysis method, based upon the equation of motion derived by Housner. Park and his researchers (1991) presented three dimensional model with 16 degrees of freedom combining velocity and pressure. They investigated the stability of piping systems with unsteady fluid flow using finite element method. And El-Rahel (1981), To and

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Kaladi (1985) and Lee and Chun (1991) used transfer matrix method in their research of piping system and calculated natural frequencies for various fluid velocities. Everstine (1986) discussed the advantage of finite elements and the feasibility of beam element for the analysis of piping structures. Finite element method is the most versatile method for the analysis of very complicated structures up to now. It requires large amount of computations and time. Due to the large size of the governing equation, it is hard to find and control design parameters using finite element method. However, the method of transfer matrices, composed of the products of point and field matrices, doesn't need large size governing equation, much computation, and time. Therefore, it is easy for the transfer matrix method to be adapted for industrial applications. Using transfer matrices, many researchers such as El-Raheb (1981), To et. al. (1985), Lee and Chun (1991) considered the effect of fluid flow only on flexural vibration, not on longitudinal vibration. Lenmez et. al. (1990) investigated the flow effect on the longitudinal vibration, not on flexural vibration. They studied on the dynamic behaviors with transient vibration of inside fluid. In the presented papers, even if transfer matrices were represented in the forms of derivatives and exponents, every degree of freedom with the effect of fluid flow in 3-dimensional space were not considered in the same time, which can be hardly used in direct for the pipe systems of complicated geometry. In this study, to overcome the difficulty in large computation with finite elements and to investigate every degree of freedom vibrations of pipe system of very complicate geometry in 3-dimensional space, overall transfer matrix are presented for three dimensional vibration with 14 DOF per node. The effects of fluid flow on the longitudinal and flexural vibrations were considered , which are related to circumferential strain and axial strain in longitudinal vibration and tension and inertia force in flexural vibration. Overall transfer matrix can be easily used for the three dimensional vibration analysis of piping systems with fluid flow. In this study,

1) Equation of motion was obtained from the relation between structure and flow.

2) Using 14 state vectors, degrees of freedom of piping structure and fluid flow were represented.

3) Transfer matrices were derived from the equation of motion.

4) Computer program was developed for the prediction and analysis of dynamic behavior of piping systems using a small size personal computer with core memory 200kbyte.

To verify the scheme of this study, the results were compared with the confirmed results using straight and curved pipes.

2. Transfer Matrix Modelling

Since the fluid flow inside of pipe line effects on the dynamic characteristics of the structure such as natural frequencies and vibrational responses, the relation between the pipe and the fluid flow should be defined and considered. The relation can be defined using force and deformation which were shown in Fig. 1. Each node has 14 degrees of freedom, 3 forces, 3 moments, 3 translations, 3 rotations, pressure and displacement of fluid. u, f, m, θ, P, V represents translational displacement, force, moment, rotation, fluid pressure and fluid displacement, respectively, t_p, r, l mean thickness, radius, length of pipe, and subscript x, y, z indicate coordinates.

2.1 Longitudinal vibration

Equations of motion for longitudinal vibration of pipe line conveying fluid can be represented as simultaneous differential equations, where there are 4 unknowns, f_x, P, V and u_x, with 4 equations

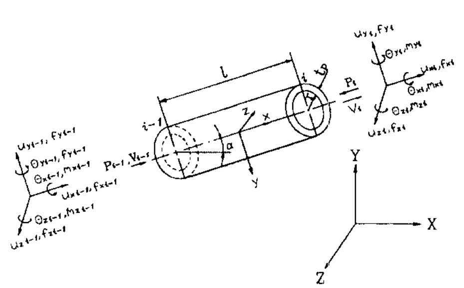

Fig. 1 Forces and displacements of pipe element

Axial force : $a^2_p\frac{\partial^2 f_x}{\partial x^2}-\frac{\partial^2 f_x}{\partial t^2}+\nu A_p b\frac{\partial^2 P}{\partial t^2}=0$ (1)

Pressure : $a^2_f\frac{\partial^2 P}{\partial x^2}-\frac{\partial^2 P}{\partial t^2}=0$ (2)

Displacement of pipe : $a^2_p\frac{\partial^2 u_x}{\partial x^2}-\nu a^2_f\frac{b}{d}\frac{\partial^2 V}{\partial x^2}-\frac{\partial^2 u_x}{\partial t^2}=0$ (3)

Displacement of fluid : $a^2_f\frac{\partial^2 V}{\partial x^2}-\frac{\partial^2 V}{\partial t^2}=0$ (4)

where

$$a^2_p=\frac{E}{\rho_p},\ b=\frac{r}{t_p},\ a^2_f=\frac{K^*}{\rho_f},\ d=\frac{\rho_p}{\rho_f}$$

$$K^*=\frac{K}{1+(K/E)(r/t_p)c_1},\ c_1=1-\frac{\nu}{2}$$

A is cross sectional area and ρ is density. Subscript p and f indicate pipe and fluid. a_p^2 and a_f^2 are wave velocities of pipe and fluid, respectively. σ, ν, E mean stress, Poisson's ratio, Young's modulus, and K is volume modulus.

Using separable variables, the solutions of the equations of motion will be

$$\begin{aligned} f_x(x,t)&=\hat{f}_x(x)e^{jwt}\\ P(x,t)&=\hat{P}(x)e^{jwt}\\ V(x,t)&=\hat{V}(x)e^{jwt}\\ u_x(x,t)&=\hat{u}_x(x)e^{jwt} \end{aligned}\qquad(5)$$

ω is circular frequency, t is time and $j=\sqrt{-1}$. Superscribe ^ is amplitude of each state vector. By substituting Eq. (5) into Eqs. (1) and (2), axial force can be represented as

$$\hat{f}_x^{iv}+\frac{(\sigma+\tau)}{l^2}\hat{f}_x''+\frac{\sigma\tau}{l^4}\hat{f}_x=0 \qquad (6)$$

where

$$\sigma=\frac{\omega^2 l^2}{a^2_p},\ \tau=\frac{\omega^2 l^2}{a^2_f}$$

Let $\hat{f}_x(x)=\bar{D}e^{\lambda\frac{x}{l}}$ be a solution of Eq. (6). Then characteristic equation will be

$$\lambda_L^4+(\tau+\sigma)\lambda_L^2+\tau\sigma=0 \qquad (7)$$

$$\lambda_{L1,2}^2=\frac{1}{2}[(\tau+\sigma)\mp\sqrt{(\tau+\sigma)^2-4\tau\sigma}] \qquad (8)$$

General solution of Eq. (6) is

$$\hat{f}_x(x)=\bar{D}_1e^{j\lambda_{L1}\frac{x}{l}}+\bar{D}_2e^{-j\lambda_{L1}\frac{x}{l}}+\bar{D}_3e^{j\lambda_{L2}\frac{x}{l}}+\bar{D}_4e^{-j\lambda_{L2}\frac{x}{l}} \qquad (9)$$

or

$$\hat{f}_x(x)=D_1\cos\left(\lambda_{L1}\frac{x}{l}\right)+D_2\sin\left(\lambda_{L1}\frac{x}{l}\right)+D_3\cos\left(\lambda_{L2}\frac{x}{l}\right)+D_4\sin\left(\lambda_{L2}\frac{x}{l}\right) \qquad (10)$$

where $\bar{D}$ and D are the arbitrary constants.

In a similar way, using separable variables, $\hat{V}(x)$, $\hat{P}(x)$ and $\hat{u}_x(x)$ are also obtained and can be represented in matrix form as

$$\{Z(x)\}=[B(x)]\{D\} \qquad (11)$$

$\{Z(x)\}$ is independent state vectors. $[B(x)]$ is a matrix which depends on material property and geometry. $\{D\}$ represents vectors for integrating constants.

where

$$\{Z(x)\}=\begin{Bmatrix}\hat{f}_x\\ \hat{P}\\ \hat{V}_f\\ \hat{u}_x\end{Bmatrix},\ \{D\}=\begin{Bmatrix}D_1\\ D_2\\ D_3\\ D_4\end{Bmatrix}$$

$$[B(x)]=\begin{bmatrix}\cos\left(\lambda_{L1}\frac{x}{l}\right) & \sin\left(\lambda_{L1}\frac{x}{l}\right) & \cos\left(\lambda_{L2}\frac{x}{l}\right) & \sin\left(\lambda_{L2}\frac{x}{l}\right)\\ B_1\cos\left(\lambda_{L1}\frac{x}{l}\right) & B_1\sin\left(\lambda_{L1}\frac{x}{l}\right) & B_2\cos\left(\lambda_{L2}\frac{x}{l}\right) & B_2\sin\left(\lambda_{L2}\frac{x}{l}\right)\\ -B_3\sin\left(\lambda_{L1}\frac{x}{l}\right) & B_3\cos\left(\lambda_{L1}\frac{x}{l}\right) & -B_4\sin\left(\lambda_{L2}\frac{x}{l}\right) & B_4\cos\left(\lambda_{L2}\frac{x}{l}\right)\\ B_5\sin\left(\lambda_{L1}\frac{x}{l}\right) & -B_5\cos\left(\lambda_{L1}\frac{x}{l}\right) & B_6\sin\left(\lambda_{L2}\frac{x}{l}\right) & -B_6\sin\left(\lambda_{L2}\frac{x}{l}\right)\end{bmatrix}$$

$$B_1=\frac{\sigma-\lambda_{L1}^2}{A_p\nu b\sigma},\ B_2=\frac{\sigma-\lambda_{L2}^2}{A_p\nu b\sigma}$$

$$B_3=\frac{(\sigma-\lambda_{L1}^2)l\lambda_{L1}}{A_p\nu bK^*\tau\sigma},\ B_4=\frac{(\sigma-\lambda_{L2}^2)l\lambda_{L2}}{A_p\nu bK^*\tau\sigma}$$

$$B_5=\frac{l\lambda_{L1}}{A_pE\sigma},\ B_6=\frac{l\lambda_{L2}}{A_pE\sigma}$$

The integrating constants of Eq. (11) can be specified using boundary conditions for the configurations of structure and motion, and the specified constants are to be combined for transfer matrices. Degrees of freedom at each node are arranged for state vector and transfer matrix is formulated as products of field and point matrices. Transfer matrix is represented as a chain of adjacent state vectors, and is a function of state vectors at both ends, $x=0$ and $x=l$. As shown in Fig. 1, $\{Z(x)\}=\{Z(x)\}_{i-1}$ at $x=0$ and $\{Z(x)\}=\{Z(x)\}_i$ at $x=l$, then equation for vector $\{D\}$ can be obtained by substituting station values into Eq. (11), then following equations will be obtained, and $[T]$ is transfer matrix.

$$\{Z\}_{i-1}=[B(0)]\{D\}$$

$$\{D\}=[B(0)]^{-1}\{Z\}_{i-1}$$

$$\{Z\}_i=[B(l)]\{D\}$$

$$\{Z\}_i=[B(l)][B(0)]^{-1}\{Z\}_{i-1}=[T]\{Z\}_{i-1} \tag{12}$$

From Eq. (12), vector $\{Z\}$, which represents $\hat{f}_x$, $\hat{P}$, $\hat{V}$ and $\hat{u}_x$, at every station can be calculated by inserting from $i=0$ to $i=n$, where n is the last station number. Transfer matrix for longitudinal vibration can be found from Eq. (12), and non-dimensionalized matrix becomes

$$[T_{fx}]=\begin{bmatrix} \sigma C_2-C_0 & -\frac{\nu b}{h}\sigma C_2 & -\frac{\nu b}{h}\sigma\tau C_3 & \sigma(C_1-\sigma C_3) \\ 2\nu\sigma\tau C_3 & \tau C_2-C_0 & \tau(\tau C_3-C_1) & -2\nu\tau C_2 \\ 2\nu\sigma C_2 & C_1-\left(\tau+\frac{\sigma}{\tau}\right)C_3 & \tau C_2-C_0 & 2\nu[(\sigma+\tau)C_3-C_1] \\ -C_1+\sigma C_3 & \frac{\nu b}{h}[C_1-(\sigma+\tau)C_3] & \frac{\nu b}{h}\tau C_2 & \sigma C_2-C_0 \end{bmatrix} \tag{13}$$

Where

$$b=\frac{r}{t_p},\ h=\frac{E}{K^*},\ \Delta=[\lambda_{L1}^2-\lambda_{L2}^2]^{-1}$$

$$\lambda_{L1}^2=\frac{1}{2}[(\tau+\sigma)-\sqrt{(\tau+\sigma)^2-4\tau\sigma}]$$

$$\lambda_{L2}^2=\frac{1}{2}[(\tau+\sigma)+\sqrt{(\tau+\sigma)^2-4\tau\sigma}]$$

$$C_0=\Delta[\lambda_{L2}^2\cos(\lambda_{L1})-\lambda_{L1}^2\cos(\lambda_{L2})]$$

$$C_1=\Delta[\frac{\lambda_{L2}^2}{\lambda_{L1}}\sin(\lambda_{L1})-\frac{\lambda_{L1}^2}{\lambda_{L2}}\sin(\lambda_{L2})]$$

$$C_2=\Delta[\cos(\lambda_{L1})-\cos(\lambda_{L2})]$$

$$C_3=\Delta[\frac{1}{\lambda_{L1}}\sin(\lambda_{L1})-\frac{1}{\lambda_{L2}}\sin(\lambda_{L2})]$$

Non-dimensionalized state vector at position i is

$$\{Z\}_i=\left\{\frac{\hat{f}_x}{EA_p}\ \frac{\hat{P}}{K^*}\ \frac{\hat{V}}{l}\ \frac{\hat{u}_x}{l}\right\}^T \tag{14}$$

2.2 Flexural vibration

In case of a long thin pipe, since bending effect is dominant, the equation of motion for the flexural vibration can be represented as

$$EI_p\frac{\partial^4 u_y}{\partial x^4}+(m_fC^2+P_0A_f-F_{xt})\frac{\partial^2 u_y}{\partial x^2}+2m_fC\frac{\partial^2 u_y}{\partial x\partial t}+M\frac{\partial^2 u_y}{\partial t^2}=0 \tag{15}$$

In Eq. (15), the first term represents the elastic force; the second, the inertia force associated with the fluid following a curved path, force for pressure of fluid and initial tension force; the third, the damping term associated with the Coriolis acceleration due to the relative motion of the fluid inside the pipe which has an angular velocity and the last term, represent the inertia force due to lateral acceleration of the pipe including the fluid flow. $m_f=\rho_fA_f$, $M=\rho_fA_f+\rho_pA_p$. C is the stationary flow velocity component and P_0 is the stationary internal pressure. The initial tension force is a static force applied to the system when the system is in the static equilibrium and $F_{xt}=\frac{EA_p}{l}(u_{xi}-u_{xi-1})$. On the other hand, Coriolis damping term is skew-symmetric matrix, when the mass ratio m_f/M is less than 0.5, which does not affect largely on the natural frequency of system (Cehn, 1972; Chen, 1973; To and Kaladi, 1985). Then Eq. (15) becomes

$$EI_p\frac{\partial^4 u_y}{\partial x^4}+(m_fC^2+P_0A_f-F_{xt})\frac{\partial^2 u_y}{\partial x^2}+M\frac{\partial^2 u_y}{\partial t^2}=0 \tag{16}$$

Transverse shear force $\hat{f}_y$, rotation angle $\hat{\theta}_z$ and bending moment $\hat{m}_z$ can be represented as

$$\frac{d\hat{f}_y}{dx}=-M\omega^2\hat{u}_y \tag{17}$$

$$\hat{\theta}_z(x,\ t)=-\frac{\partial\hat{u}_y}{\partial x} \tag{18}$$

$$\hat{m}_z(x,\ t)=EI_p\frac{\partial\hat{\theta}}{\partial x} \tag{19}$$

Transfer matrix for flexural vibration can also be obtained in the same way as the longitudinal vibration, and non-dimensionalized matrix becomes

$$[T_{xy}]=\begin{bmatrix} \Delta C1 & \gamma\Delta C2 & -\gamma\Delta C3 & \gamma\Delta C4 \\ \frac{\Delta}{\gamma}\lambda_{F1}\lambda_{F2}C5 & \Delta C6 & \Delta\lambda_{F1}^2\lambda_{F2}^2C7 & \Delta C8 \\ -\frac{\Delta}{\gamma}\lambda_{F1}\lambda_{F2}C9 & -\Delta C4 & \Delta C1 & -\Delta C5 \\ \frac{\Delta}{\gamma}\lambda_{F1}\lambda_{F2}C4 & \Delta C5 & -\Delta\lambda_{F1}\lambda_{F2}C10 & \Delta C6 \end{bmatrix} \quad (20)$$

Where

$$\sigma=\frac{(m_f V^2+PA_f-F_{xt})}{EI_p}l^2,\ \gamma=\frac{M}{EI_p}\omega^2 l^4$$

$$\lambda^2_{F1}=\frac{1}{2}[\sqrt{\sigma^2+4\gamma}-\sigma]$$

$$\lambda^2_{F2}=\frac{1}{2}[\sqrt{\sigma^2+4\gamma}+\sigma]$$

$$\Delta=[\lambda_{F1}^2-\lambda_{F2}^2]^{-1}$$

$$C1=\lambda_{F2}^2\cosh\lambda_{F1}+\lambda_{F1}^2\cos\lambda_{F2}$$

$$C2=\cosh\lambda_{F1}-\sin\lambda_{F2}$$

$$C3=\frac{\lambda_{F2}^2}{\lambda_{F1}}\sinh\lambda_{F1}-\frac{\lambda_{F1}^2}{\lambda_{F2}}\sin\lambda_{F2}$$

$$C4=\cosh\lambda_{F1}-\cos\lambda_{F2}$$

$$C5=\lambda_{F1}\sinh\lambda_{F1}+\lambda_{F2}\sin\lambda_{F2}$$

$$C6=\lambda_{F1}^2\cosh\lambda_{F1}+\lambda_{F2}^2\cos\lambda_{F2}$$

$$C7=\cosh\lambda_{F1}+\cos\lambda_{F2}$$

$$C8=\lambda_{F1}^3\sinh\lambda_{F1}-\lambda_{F2}^3\sin\lambda_{F2}$$

$$C9=\lambda_{F2}\sinh\lambda_{F1}-\lambda_{F1}\sin\lambda_{F2}$$

$$C10=\lambda_{F2}\sin\lambda_{F1}+\lambda_{F1}\sin\lambda_{F2}$$

Non-dimensionalized state vector at position i is

$$\{Z\}_i=\left\{\frac{\hat{f}_y l^2}{EI_p}\ \frac{\hat{m}_z l}{EI_p}\ \frac{\hat{u}_y}{l}\ \hat{\theta}_z\right\}_i^T \quad (21)$$

Transfer matrix with respect to x-z plane is also as Eq. (20) with some negative component, and non-dimensionalized state vector at i is

$$\{Z\}_i=\left\{\frac{\hat{f}_z l^2}{EI_p}\ \frac{\hat{m}_y l}{EI_p}\ \frac{\hat{u}_z}{l}\ \hat{\theta}_z\right\}_i^T \quad (22)$$

2.3 Torsional vibration

For curved pipes, torsional vibration is not dominant, but coupled with bending vibration. With very small viscosity, the fluid effect on the torsional vibration is negligible, and the equations of motion become

$$\frac{\partial m_x}{\partial x}-\rho_p J_p\frac{\partial^2\theta_x}{\partial t^2}=0 \quad (23)$$

$$m_x-GJ_p\frac{\partial\theta_x}{\partial x}=0 \quad (24)$$

G and J_p are shear modulus and polar inertia of pipe.

Transfer matrix for torsional vibration can be obtained in the same way, and non-dimensionalized transfer matrix is

$$[T_{tx}]=\begin{bmatrix} -\cos(\lambda_T) & \lambda_T\sin(\lambda_T) \\ -\frac{1}{\lambda_T}\sin(\lambda_T) & -\cos(\lambda_T) \end{bmatrix} \quad (25)$$

Where

$$\lambda_T=\omega l\sqrt{\frac{\rho_p}{G}}$$

Non-dimensionalized state vector is

$$\{Z\}_i=\left\{\frac{\hat{m}_x l}{GJ_p}\ \hat{\theta}_x\right\}_i^T \quad (26)$$

3. Overall Transfer Matrix for Pipe Element

Transfer matrix for pipe element is composed of previously derived four matrices, and state vector contains 14 independent variables per each node.

$$\{Z\}_i=[T_L]\{Z\}_{i-1} \quad (27)$$

$[T_L]$ represents transfer matrix for pipe element with length L

$$[T_L]=\begin{bmatrix} [T_{fx}] & & & \\ & [T_{xy}] & & \\ & & [T_{xz}] & \\ & & & [T_{tx}] \end{bmatrix} \quad (28)$$

Non-dimensionalized state vector at position i is

$$\{Z\}_i=\left\{\frac{\hat{f}_x}{A_pE}\ \frac{\hat{P}}{K^*}\ \frac{\hat{V}}{l}\ \frac{\hat{u}_x}{l}\ \frac{\hat{f}_y l^2}{EI_p}\ \frac{\hat{m}_z l}{EI_p}\ \frac{\hat{u}_y}{l}\ \hat{\theta}_z\ \frac{\hat{f}_z l^2}{EI_p}\ \frac{\hat{m}_y l}{EI_p}\ \frac{\hat{u}_z}{l}\ \hat{\theta}_y\ \frac{\hat{m}_x l}{GJ_p}\ \hat{\theta}_x\right\}_i^T \quad (29)$$

Overall transfer matrix for piping system is a function of ω, and is formulated through 3 steps. Arrange the matrices, transform the arranged matrices, which are described with respect to local coordinate system into global coordinate system, and product the transformed matrices successively.

4. Natural Frequency and Critical Flow Velocity

Natural frequency of piping systems is one of very important factors in design, and the size of frequency equations depends on boundary conditions.

$$\text{Boundary Condition of Left} = [R_L] \tag{30}$$
$$\text{Boundary Condition of Right} = [R_R] \tag{31}$$

Some typical boundary conditions are:

Hinged-Hinged : $P = u_x = u_y = m_z = 0$, at $x = 0$
$P = u_x = u_y = m_z = 0$, at $x = l$

Clamped-Clamped : $P = u_x = u_y = \theta_z = 0$, at $x = 0$
$P = u_x = u_y = \theta_z = 0$, at $x = l$

Equations (30) and (31) is 14×14 matrices, and the size can be reduced according to the motion of the piping systems. The relationships between transfer matrix and state vector can be described by Eq. (32). Subscripts 0 and 1 represent left and right positions, respectively.

$$\{Z\}_1 = [U]\{Z\}_0 \tag{32}$$

Overall transfer matrix is related to boundary conditions at both ends, applying boundary conditions.

$$\{0\} = [R_R][U][R_L]\{Z\}_0$$

or

$$\{0\} = [U_G]\{Z\}_0 \tag{33}$$

Vector $\{Z\}_0$ represents non-zero vector, and the size of matrix $[U_G]$ can be reduced using boundary conditions. For Eq.(33) to have non-trivial solution,

$$\Delta = |[U_G]| = 0 \tag{34}$$

Eq.(34) is called frequency equation and Δ is called frequency determinant. The values of ω which satisfy the frequency equation is natural frequencies of the system. In piping systems, as fluid velocity increases, the values of natural frequencies decrease. When the calculated value of natural frequency becomes zero, buckling takes place and the system loses stability. Such a velocity that natural frequency becomes zero is called as critical velocity and can be calculated from Eq. (34) as a fundamental eigenvalue.

5. Result and Discussion

To confirm the scheme in this study, computer program was developed based on the transfer matrices formulated in the previous section, and the calculated values were compared with the known data from references Blevins (1990), Kohli and Nakr (1984) and Park et al. (1991). In this study, natural frequencies with different flow velocities are investigated for fluid-filled straight and curved pipes conveying steady state fluid flow. Since flow frequencies are much smaller than the fundamental natural frequency of pipe structures, the resonance effect was ignored. In the case of curved pipes, both in-plane and out-of-plane vibration have been considered. In this study, each pipe was divided as seven nodes and calculated to get the coincided results. With different nodes the calculated frequencies have almost same values in low frequency range, 1st, 2nd, 3rd frequencies, but in high frequency range, there were discrepancies in the calculated values, considered as the shear effect. Table 1 shows physical

Table 1 Physical parameters of pipe

Young's modulus	208 Gpa
Poisson ratio	0. 3
Density of pipe	8000Kg/m^3
Density of fluid	1000Kg/m^3
Bulke modulus of fluid	2.19Gpa
Pipe outside diameter	9.54×10^{-3}m
Pipe wall thickness	1×10^{-3}m
Pipe length, straight	0.5m
Pipe radius, curved	0.125m

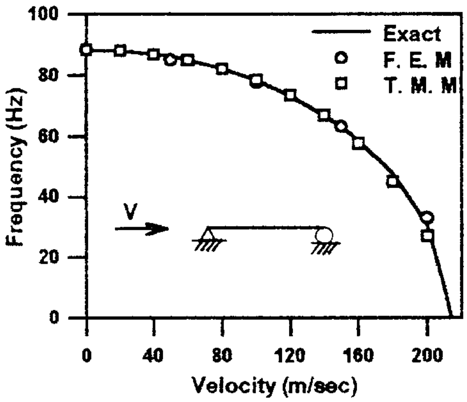

Fig. 2 The lowest natural frequency of a straight pipe

Table 2 Physical parameters of pipe

Young's modulus	124.11Gpa
Shear modulus	46.197Gpa
Poisson ratio	0.34
Density of pipe	$8900 Kg/m^3$
Density of fluid	$1000 Kg/m^3$
Bulke modulus of fluid	2.19Gpa
Pipe outside diameter	19.05×10^{-3}m
Pipe wall thickness	0.90×10^{-3}m
Pipe length	2.4m

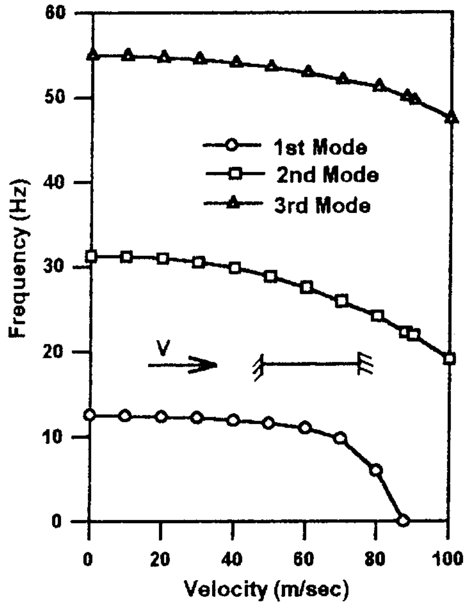

Fig. 3 Natural frequencies for normal modes of a straight pipe

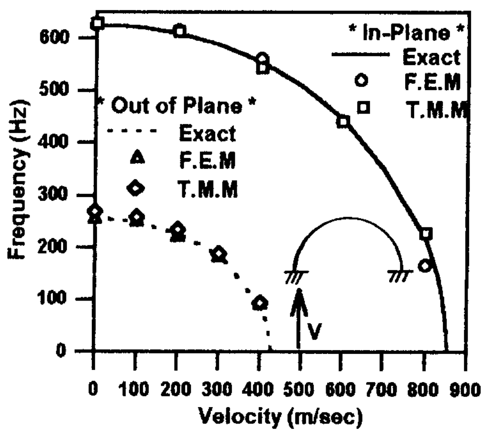

Fig. 4 The lowest natural frequency of a curved pipe

parameters of simply supported straight pipe, of which fundamental natural frequency was calculated using the program developed in this study.

The results were compared with those of finite element method by Kohili and Nakra (1984) and analytic method presented by Blevins (1990), they agreed well as shown in Fig. 2. T. M. M. means transfer matrix method, which was adapted in this study. The physical parameters of clamped-clamped straight pipe are shown in Table 2 and its natural frequencies vs. flow velocity is shown in Fig. 3. The fundamental frequency at zero flow velocity was 12.3Hz(77rad/sec), which was the same value from analytic method by Blevins (1990), when the flow velocity was small enough or zero. Near critical velocity, the fundamental frequency becomes zero by buckling.

In-plane vibration of a curved pipe includes longitudinal and flexural modes, out-of-plane vibration includes torsional mode in addition to in-plane modes. For curved pipe with $\theta=90°$, 135° and half ring segment type, the relations between natural frequency, flow velocity, and curved angle were investigated and the critical velocities were calculated. Half ring segment type of curved pipe was analyzed using the scheme in this study with parameters in Table 1. In Fig. 4, trend of fundamental natural frequencies vs. flow velocities is shown and the calculated values were compared with those of different methods. In-plane natural frequencies were very coincident with the result by Kohli and Nakra when flow velocities were zero or very small, but out-of-plane natural frequencies were calculated to be 5% higher than the value by finite element method

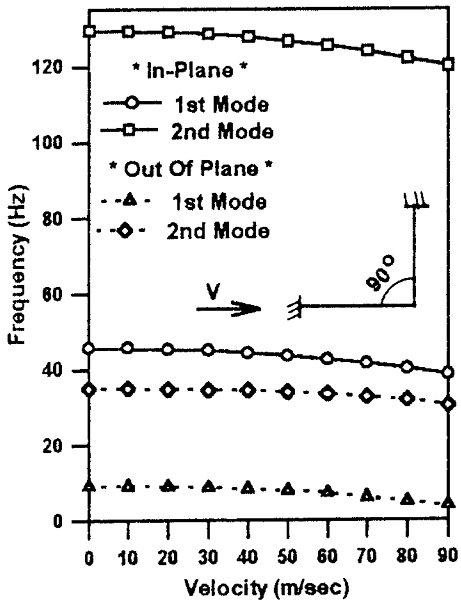

Fig. 5 Natural frequencies for normal modes of a curved pipe

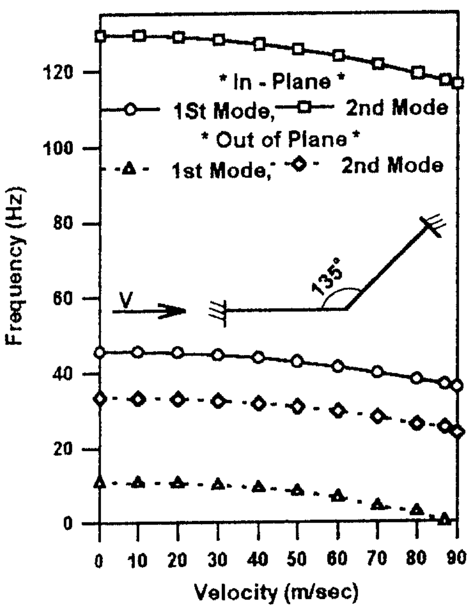

Fig. 6 Natural frequencies for normal modes of a curved pipe

by Kohli and Nakra's (1984) study. The reason was guessed that initial curvature was not considered in finite element method. With parameter values as in Table 2, curved pipes with curved angles 90° and 135° as in Park's et. al. (1991) paper were analyzed and their natural frequencies were calculated, and represented in Figs. 5 and 6 respectively. When flow velocity was small enough or zero, in-plane fundamental natural frequencies were not changed, even if the curved angles were changed, but out-of-plane natural frequencies were found to be changed. Out-of-plane natural frequencies were 6.3Hz(57rad/sec) and 11.0Hz(68.8rad/sec) for $\theta=90°$ and 135°, respectively, which means that the pipe of 135° curved angle is stiffer than the pipe of right curved angle. Consequently, the facts were observed that the values of natural frequencies rose with increasing curved angles and dropped with increasing flow velocities. Large differences in values of natural frequencies between in-plane and out-of-plane modes were due to the effect of initial tension of fluid flow on the out-of-plane mode. Near critical velocity, curved pipes fell in unstable condition as in straight pipes by buckling.

Critical velocities in out-of plane motion of clamped-clamped curved pipes of different angles, with physical parameters as shown in Table 2, were calculated and represented in Table

Table 3. Critical velocity of a clamped-clamped pipe

Angle(θ°)	Critical Velocity(m/sec)
180°	87.74
170°	87.93
160°	87.02
150°	86.41
140°	86.34
130°	87.87
120°	92.73
110°	104.70
100°	138.00
95°	189.25

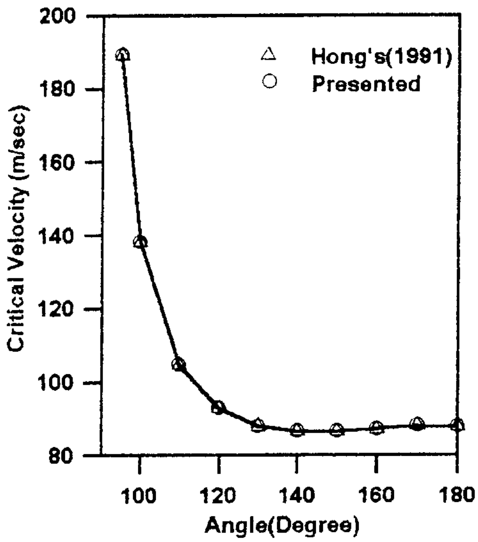

Fig. 7 Critical velocity of a clamped pipe

3. The results were compared with the results of Hong's paper (1991) and they agree very well as shown in Fig. 7. The values, which depend upon boundary conditions and geometric shapes, increase with decrease of the angles for the same conditions. For the right angle, critical velocity approaches to the infinite value, which means there is no sudden drop of natural frequency. The reason is known that tension force resists centrifugal force in equilibrium.

6. Conclusions

Piping systems with fluid flow, modelled as three dimensional shapes were analyzed for dynamic behaviors such as longitudinal and flexural vibrations due to the fluid velocity and pressure, which were neglected in previous studies. Based upon the equation of motion for three dimensional dynamic behaviors, transfer matrices were formulated, and a computer algorithm was presented. The scheme in this study can be implemented on a small size personal computers, and easily be adapted for practical applications in another fields. Some results obtained are as follows.

① Transfer matrices formulated in this study can be applied to any types of pipes of arbitrary shapes with elbow parts, even if masses or supports are complicatedly attached to the pipe lines.

② Dynamic characteristics were dependent largely on physical parameters of pipes rather than on fluid flow properties. Especially bending effects were found to be dominant.

③ The calculated natural frequencies were compared with the values of previously presented works and analytic method. The error was less 10% within the range of $V \leq Vcr$. As flow velocity increased, the fundamental natural frequency decreased, finally the system fell in unstable condition when the natural frequency dropped suddenly to zero, at critical velocity.

④ Consequently, in the design of piping system with elbows, out of plane vibration should be taken into account more precisely than in-plane vibration.

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